P. O. TJØM AND B. ELBEK

A STUDY OF ENERGY LEVELS IN ODD-MASS ERBIUM NUCLEI BY MEANS OF (*d,p*) AND (*d,t*) REACTIONS

Det Kongelige Danske Videnskabernes Selskab
Matematisk-fysiske Meddelelser **37**, 7

Kommissionær: Munksgaard
København 1969

CONTENTS

Synopsis

The energy levels of ^{161}Er, ^{163}Er, ^{165}Er, ^{167}Er, ^{169}Er and ^{171}Er have been investigated by means of (d,p) and (d,t) reactions on the stable erbium isotopes. The deuteron energy was 12.1 MeV and the charged reaction products were analyzed in a magnetic spectrograph at 60°, 90°, and 125°. A total of 16 different Nilsson orbitals or components thereof were identified on the basis of the intensity patterns for the rotational states, the absolute cross sections, and the rate of intensity change with angle. For the majority of the orbitals, the observations are in reasonable agreement with the theoretical predictions based on the single-particle functions in a deformed potential. A few of the observed intensities do, however, deviate considerably from the theoretical intensities. Among the reasons for such deviations are the crossing of energy levels from different oscillator shells and couplings to other single-particle states or collective vibrations, but for a number of cases no obvious explanation has yet been found.

PRINTED IN DENMARK
BIANCO LUNOS BOGTRYKKERI A/S

1. Introduction

The present study of the energy levels in the odd erbium isotopes by means of neutron stripping and pick-up reactions is a continuation of earlier investigations of the energy levels in odd gadolinium[1] and ytterbium[2] nuclei.

The main motivation for this type of experiments is the possibility it offers for a systematic localization of a large number of neutron single-particle states. At the same time, the amplitudes of certain components of the wave functions are obtained from the observed single-particle transfer cross sections. The interpretation of the cross sections is based on relatively few assumptions about the nuclear reaction mechanism and has been tested in a large number of cases.

A complex structure of the wave function of the excited nuclear states often complicates the analysis of the transfer reaction data and, although there has been little reason to doubt the single-nucleon nature of the (d,p) and (d,t) reaction studies, there are also in the present work several examples of observations, which cannot be accounted for in a satisfactory manner.

Among the phenomena which limit the applicability of the single-nucleon description is the particle coupling to the various collective vibrational modes. Examples of such couplings have been discussed in earlier papers[1, 2], especially in connection with the gamma vibrational states. The even erbium nuclei have low-lying vibrations, which are connected with the ground states by large $E2$ matrix elements. These and other collective states in the even nuclei have been studied by means of the (d,d') reaction[3], actually on the basis of the charged particle spectra of which the proton and triton parts are analyzed here. The well-developed gamma vibrational states in the erbium nuclei offer a possibility for the study of the particle-vibration coupling in deformed nuclei. Most of the cases investigated up to now have been characterized by relatively weak collective states, and it is an interesting problem whether the large single-particle amplitudes in the vibrational states[1, 2] in the odd nuclei are also observed when the vibrations are strongly collective.

Probably, the Coriolis coupling between rotational bands differing one unit in the angular momentum projection, K, is the most important and well understood phenomenon, which gives rise to intermixing of the one-particle wave functions. The Coriolis coupling is obviously responsible for a large fraction of the observed departures from the simple theoretical intensity distributions for the one-nucleon transfer reactions. However, only in a few cases has the material available been subject to a detailed analysis of such effects.

The coupling between single-particle states with $N = 4$ and $N = 6$ was found to be of major importance for several spectra of the gadolinium nuclei. The single-particle states in question also occur as relatively low-lying levels in the erbium nuclei and thus permit a further study of this type of coupling.

The experimental methods are closely the same as those used before[1, 2]. The beam of 12.1 MeV deuterons was obtained from the Niels Bohr Institute's tandem accelerator, and the charged reaction products were analyzed in a high-resolution magnetic spectrograph with photographic plate recording. The targets for the investigations were $\sim 40\ \mu g/cm^2$ layers of the relevant isotope directly deposited on $\sim 40\ \mu g/cm^2$ carbon foils in the electromagnetic isotope separator at the University of Aarhus.

The absolute spectroscopic factors obtained from the (d,p) and (d,t) cross sections depend in a critical manner on the nuclear optical parameters used for the reaction calculations by means of the distorted wave Born approximation (DWBA) method. It has been the general philosophy followed in the earlier investigations first to select a set of reasonable potentials and, then, to use these potentials throughout. In this way, no optimum adjustment to angular distribution data is obtained, but, on the other hand, the important comparison of spectroscopic factors for the different nuclei is facilitated. Moreover, the limitations of the fixed potentials are not easily realized, as there is a lack of detailed angular distribution data. Unfortunately, the deuteron potential selected originally[2] was somewhat shallow compared to the standard potential of Perey[4]; nevertheless, it gave satisfactory results for the (d,t) angular distributions with minor adjustments of the triton parameters[5]. Also the (d,p) angular distributions were satisfactory, although little experimental material was available for comparison[6, 7]. When the same parameters were used for the Er nuclei, the calculated angular distributions for the (d,t) reactions were essentially unchanged, but the (d,p) distributions—especially for even l-values— showed pronounced oscillations, which have not been observed experimentally in the few cases investigated. No (d,p) distribution for even l has been measured in Er, but

TABLE 1. The DWBA single-particle cross sections $\sigma_l(90^\circ)$ for (d,t) and (d,p) reactions.

	$\sigma_0(90^\circ)$	$\sigma_2(90^\circ)$	$\sigma_4(90^\circ)$	$\sigma_1(90^\circ)$	$\sigma_3(90^\circ)$	$\sigma_5(90^\circ)$	$\sigma_0(90^\circ)$	$\sigma_2(90^\circ)$	$\sigma_4(90^\circ)$	$\sigma_6(90^\circ)$
Reaction	$N = 4$ $\mu b/sr$	$N = 4$ $\mu b/sr$	$N = 4$ $\mu b/sr$	$N = 5$ $\mu b/sr$	$N = 5$ $\mu b/sr$	$N = 5$ $\mu b/sr$	$N = 6$ $\mu b/sr$	$N = 6$ $\mu b/sr$	$N = 6$ $\mu b/sr$	$N = 6$ $\mu b/sr$
(d,t) $Q = -2$ MeV	214	120	23.4	251	88.4	13.7	372	222	58.5	6.4
(d,p) $Q = 3$ MeV				500	195	26.5	580	365	102	15.5

The optical model parameters used for the calculation are those listed in Table 1 of ref. 1, which also defines the DWBA cross section $\sigma_l(\theta)$.

it seems unlikely that the experimental distributions should show oscillations as pronounced as those calculated. Somewhat arbitrarily, a smooth curve was drawn to reproduce the main trends in the calculated distributions. This procedure found some justification in the fact that a calculation based on the standard deuteron parameters considerably reduces the oscillations without significant changes in the absolute cross sections. Obviously, this point needs clarification; however, in order to be consistent with earlier spectroscopic factor calculations, the above-mentioned averaging procedure was used. The DWBA single-particle cross sections $\sigma_l(\theta)$ (defined as in ref.[1]) for the reference Q-values $+3$ MeV for (d,p) and -2 MeV for (d,t) are listed in Table 1.

2. Results and Discussion

In Figs. 1–10 a spectrum is shown for each of the ten different transfer reactions possible with the stable targets ^{162}Er, ^{164}Er, ^{166}Er, ^{168}Er, and ^{170}Er. The level energies obtained as averages of the determinations at three different angles are listed in Tables 2–7, which also contain the measured differential cross sections and the suggested Nilsson assignments for some of the levels. The basis for these assignments will be discussed in detail in the following sections.

2.1. *Q*-values

The identification of the ground-state group did not cause any problems, except in the case of ^{167}Er where the ground-state group was weak. The ground-state Q-values were therefore based on an excitation energy of

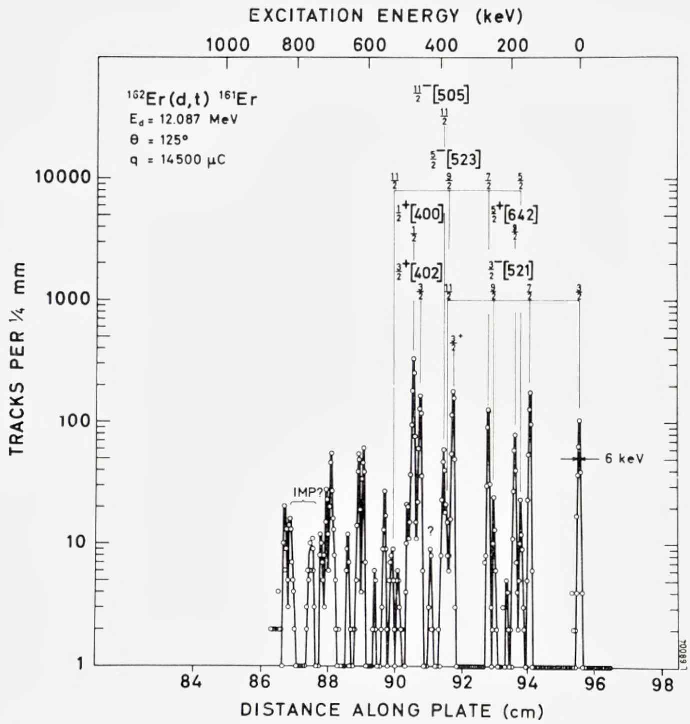

Fig. 1. Triton spectrum for the reaction ^{162}Er$(d,t)^{161}$Er $\theta = 125°$.

79 keV for the well-known 9/2 7/2 + [633] state. The final Q-values, corrected for small effects from partial magnetic saturation of the spectrograph iron, are given in Table 8, which also lists the neutron separation energies derived from the Q-values.

2.2. General Features of the Spectra

The (d, t) spectra were scanned from the ground state to the position of the elastic deuteron group. In the heaviest isotopes, this corresponds to a region of excitation of about 2 MeV, in the lightest to about 800 keV. The energy resolution in the (d, t) spectra was about 6 keV, which in most cases was sufficient to ensure well-separated groups. The (d, p) spectra were scanned up to 2 MeV of excitation. Because of the high proton energy (15 MeV) and the lower spectrograph dispersion at the smaller radii of curvature at which the proton spectra were recorded, the energy resolution was only

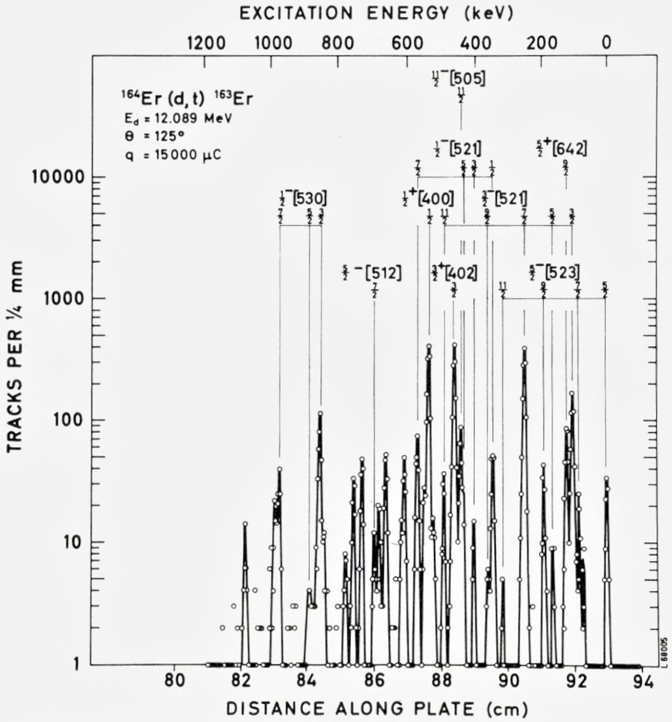

Fig. 2. Triton spectrum for the reaction ^{164}Er$(d,t)^{163}$Er $\theta = 125°$.

about 12 keV. At higher energy of excitation, this was insufficient to ensure complete separation of the proton groups.

The density of levels populated by the transfer reactions shows a moderate increase with excitation energy. Especially for the lighter isotopes, the number of levels populated by the (d,p) reaction is quite large in the region above 1 MeV of excitation. In the same region, the intensities tend to be more evenly distributed, so that the spectra lack easily recognizable patterns (compare, e.g. Fig. 3 and Fig. 10).

The level schemes Figs. 12–17 show that, in general, it has been possible to make rather definite assignments for most of the levels below 1 MeV although, in the lighter isotopes, also some of the lower levels remain unassigned. Already here it should, however, be stressed that the assignments only imply that a sizable fraction of the total single-particle strength of a given Nilsson orbital is found at the positions indicated.

In the earlier investigation of the gadolinium isotopes, it was shown

TABLE 2. Levels populated in ^{161}Er.

Energy average (d,t) keV	Assignment	$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
		60°	90°	125°
0	3/2 3/2 – [521]		55	32
144	7/2 3/2 – [521]	45	77	59
172	5/2 5/2 – [523]	6	9	8
189	9/2 5/2 + [642] ?	26	34	28
212		5	5	~ 2
250	9/2 3/2 – [521]	2	7	7
268	(7/2 5/2 – [523])	26	50	50
369	3/2 3/2 + [402]*	36	79	70
388	{ 11/2 3/2 – [521] 9/2 5/2 – [523] }	2	5	6
396	11/2 11/2 – [505]	5	20	23
463	3/2 3/2 + [402]*	40	85	64
481	1/2 1/2 + [400]	73	143	118
495		8	10	7
522		4	4	3
540	11/2 5/2 – [523]		3	5
563		4	13	10
588				2
621		10	21	23
635		16	27	22
665		3	4	4
704		4	10	4
712		12	23	20
724		5	10	10
738		4	8	5
842		5	7	5

* Splitting of intensity probably caused by interaction with 3/2 + [651].

that most of the strength expected on the basis of the Nilsson model was present. A similar analysis for the erbium isotopes confirms this statement especially as far as the hole states are concerned. The strength of the particle states is somewhat less than expected, the total (d,p) cross sections to levels below 2 MeV of excitation being about 75 % of the theoretical value. It is not clear whether this reflects discrepancies in the theoretical cross sections used for the comparison or whether some of the strength has been pushed to higher energies.

TABLE 3. Levels populated in ^{163}Er.

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
0	0	5/2 5/2 − [523]	18	11		14	20	11
~ 67	~ 69	7/2 5/2 + [642]?		~ 1	~ 2	~ 4		~ 2
82	84	7/2 5/2 − [523]	28	19	6	8		7
102	104	3/2 3/2 − [521]	71	34	8	87	101	60
119	121	9/2 5/2 + [642]	32	25	13	36	53	33
~ 159	164	5/2 3/2 − [521]		~ 1		2		2
188	190	9/2 5/2 − [523]	19	19	10	7	18	14
247	250	7/2 3/2 − [521]	185	153	94	120	198	145
	320	11/2 5/2 − [523]				~ 1	~ 1	~ 1
344	344	1/2 1/2 − [521]	232	103	45	40	47	22
	359	9/2 3/2 − [521]				3	5	2
404	405	3/2 1/2 − [521]	26	23	10	11	11	4
439		5/2 1/2 − [521]		38	21			
	444	11/2 11/2 − [505]				15	39	33
461	463	3/2 3/2 + [402]	~ 50	33	16	108	172	147
495	495	11/2 3/2 − [521]	6	6	9	2	7	13
	525						3	7
541	541	1/2 1/2 + [400]	79	35	16	126	207	156
	553					6	13	8
570	573	7/2 1/2 − [521]	84	64	29	16	34	26
609	610	5/2 5/2 − [512]	10	7	5	9	15	17
	619					2	5	4
636		9/2 1/2 − [521]	10	8	6			
	664					14	26	20
	683					9	11	7
699	698	7/2 5/2 − [512]	256	156	69	3	5	3
	735					7	17	17
757	759		14	6	2	12	18	12
779	781		12	11	6	3	5	3
~ 805		9/2 5/2 − [512]	~ 1	~ 2	~ 1			
827			13	10	10			
841	842		27	28	10	5	4	4
854	856	3/2 1/2 − [530]	42	21	7	33	63	37
872	877	5/2 1/2 − [530]	7	8	4	2	4	~ 1
	973	7/2 1/2 − [530]				8	16	13
979			21	16	12			
	987					4	10	8
1029			6	3	2			
1055			12	5	4			
1074	1075	1/2 1/2 − [510]	8	5	~ 1		4	4

(continued)

TABLE 3 (continued).

Energy average (d,p) keV	Energy average (d,t) keV	Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$ 60°	90°	125°	$d\sigma/d\Omega(d,t)$ $\mu b/sr$ 60°	90°	125°
1098		3/2 1/2 – [510]	246	143	59			
1164			11	15	8			
1183		5/2 1/2 – [510]	59	44	26			
1204			11	7	4			
1245		7/2 1/2 – [510]	44	19	7			
1277			40	24	12			
1316			35	31	6			
1344			88	79	33			
1395		9/2 1/2 – [510]	10	5	2			
1433			68	40	17			
1485			87	52	20			
1529			80	40	19			
1562			76	45	24			
1635			40	22	7			
1671			45	19	13			
1686			44	37	18			
1717			28	12	7			
1759			80	41	21			
1784			33	25	11			
1803			46	23	10			
1817			37	21	14			
1856			34	18	7			
1871			27	12	6			
1900			34	18	12			
1920			49	30	13			
1938			89	49	19			
1959			58	29	11			
1971			24	22	8			
1984			20	13	4			
2019			75	40	24			
2031			76	38	16			
2051			73	51	26			
2077			140	102	58			
2096			30	27	13			
2113			67	36	19			
2135			57	37	20			
2148			43	34	17			
2165			45	29	22			
2183			54	34	20			
2200			36	22	8			

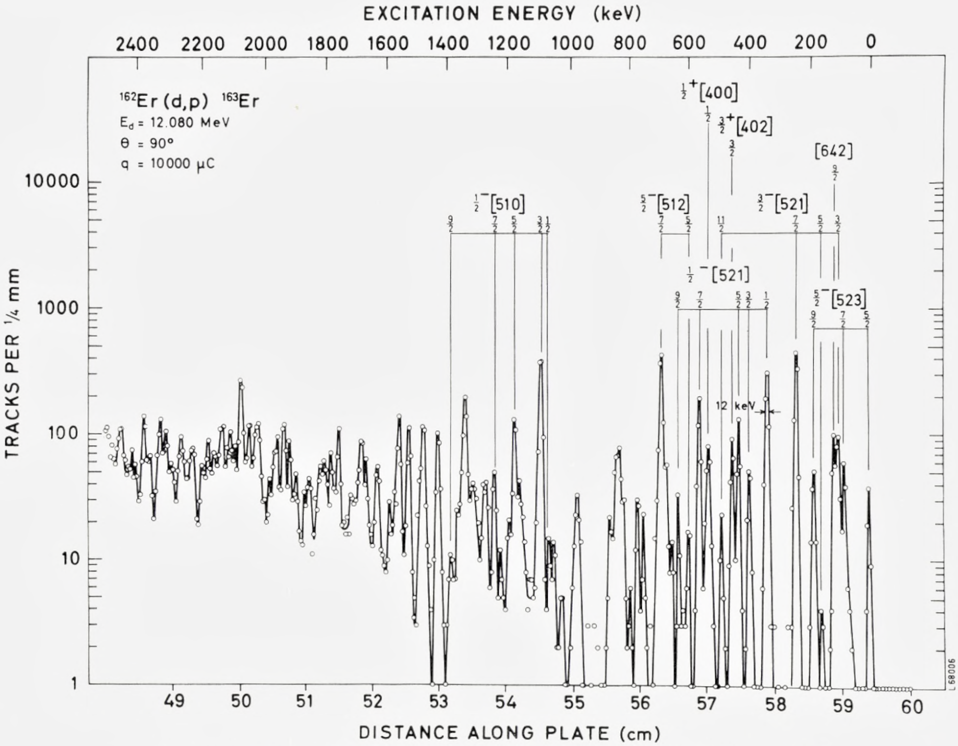

Fig. 3. Proton spectrum for the reaction ^{162}Er$(d,p)^{163}$Er $\theta = 90°$.

2.3. Detailed Interpretation of the Spectra

The methods of interpretation closely follow those used for gadolinium and ytterbium. The discussion below is arranged according to the Nilsson assignments for the different bands identified. As remarked earlier, an $IK\pi[Nn_z\Lambda]$ assignment indicates only that the Nilsson orbital in question contributes an essential fraction of the wave function. In a number of cases, it has been possible to identify some of the couplings responsible for the splitting of the single-particle intensity among several bands. These cases are discussed under the heading of that single-particle level which receives most of the intensity.

2.3.1. The 3/2-[521] Orbital

The 3/2 – [521] orbital was known previously[8, 9] in the isotopes from ^{161}Er to ^{167}Er. The present assignments are in agreement with the earlier

TABLE 4. Levels populated in ^{165}Er.

Energy average (d,p) keV	Energy average (d,t) keV	Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$ 60°	90°	125°	$d\sigma/d\Omega(d,t)$ $\mu b/sr$ 60°	90°	125°
0	0	5/2 5/2 – [523]	9	6		30	34	17
	48	5/2 5/2 + [642]				8	3	2
76	76	7/2 5/2 – [523]		7	3	11	16	9
99	98	9/2 5/2 + [642]		21	8	53	71	37
176	176	9/2 5/2 – [523]	13	11	5	15	33	27
240	242	3/2 3/2 – [521]	80	57	44	159	233	124
298	297	1/2 1/2 – [521]	256	121	48	92	92	39
356	355	3/2 1/2 – [521]	8	13	6	12	9	5
373	372	7/2 3/2 – [521]	198	122	44	164	217	136
~ 382	~ 384	5/2 1/2 – [521]		27	21	~ 13	~ 21	~ 13
470	469	9/2 3/2 – [521]		4	2	2	~ 5	5
	507	1/2 1/2 + [660]				114	~ 168	102
514		7/2 1/2 – [521]	89	60	27			
533	534	3/2 3/2 + [402]	43	27	14	169	~ 305	
	547					~ 17	~ 57	~ 36
575	575	7/2 5/2 – [512]	288	175	83	~ 10	~ 18	13
593	591	11/2 11/2 – [505]		42	17	~ 18	~ 33	~ 27
608	601		26	18	5	~ 10	~ 23	~ 9
	652						3	6
684		9/2 5/2 – [512]		7	3			
700			9	5	3			
728	724		~ 29	24	9	5		5
746	742	1/2 1/2 + [400]	~ 13	7	5	114	190	139
761	760			6	3	15	40	24
820	817	11/2 5/2 – [512]		7	2	30	58	32
846				4	4			
	863					14	17	6
873				10	5			
896			6	8	5			
925		1/2 1/2 – [510]		~ 2	~ 1			
961		3/2 1/2 – [510]	205	119	46			
	972					9	14	13
1024		5/2 1/2 – [510]	74	48	28			
	1039	3/2 1/2 – [530]				65	96	56
1043			70	42	10			
	1063	5/2 1/2 – [530]				4	8	5
1073			9	11	2			
1110		7/2 1/2 – [510]	30	31	10			
	1107					8	9	6
	1139					9	15	9

(continued)

TABLE 4 (continued).

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
1145			6	4	6			
	1172	7/2 1/2 – [530]				6	17	14
1177			14	5	6			
1198			5	3	5			
1233			4	7	6			
	1276					18	28	24
1285			114	82	36			
1378			14	14	9			
	1383					10	13	12
1413			65	35	14			
1474		3/2 3/2 – [512]	99	49	22			
1490				14	5			
1539		5/2 3/2 – [512]	123	82	44			
1564			44	21	7			
1612			39	~ 28	7			
1631		7/2 3/2 – [512]	56	~ 28	13			
1656			79	63	26			
1728			77	44	9			
1761			112	51	22			
1780			66	45	16			
1805			83	45	21			
1819			102	63	25			
1851			56	23	14			
1889			65	35	20			
1901			90	53	22			
1940			173	78	47			
1951				38	33			
1968			176	86	39			
2004			33	22	7			
2018				23	11			
2033			107	45	11			
2047				35	24			
2057			75	30	23			

ones. In ^{169}Er, there is a pattern similar to that observed for the 3/2 – [521] orbital in the other nuclei with a band-head energy of 713 keV. The 3/2 –, 7/2, and 9/2 – members of the band are observed in all the erbium isotopes except ^{171}Er, where the (d,p) spectra do not allow any identification of the 3/2 – [521] band, which here occurs as a hole excitation. In addition, the

TABLE 5. Levels populated in ^{167}Er.

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
0	0	7/2 7/2 + [633]	~ 1		~ 0.3	~ 2	~ 1	~ 0.6
79	79	9/2 7/2 + [633]	19	9	8	41	57	22
176	177	11/2 7/2 + [633]	~ 3		~ 1	~ 1	~ 2	~ 1
208	208	1/2 1/2 – [521]	292	149	51	265	201	72
262	264	3/2 1/2 – [521]	5	10	4	9	5	3
280	281	5/2 1/2 – [521]	63	38	18	32	34	16
295	295	13/2 7/2 + [633]	27	42	34	37	71	50
347	345	5/2 5/2 – [512]	13		3	2	1	~ 0.5
413	414	7/2 1/2 – [521]	125	84	37	60	65	37
430	431	7/2 5/2 – [512]	304	260	112	66	82	41
	~ 438	9/2 1/2 – [521]						~ 8
535	534	9/2 5/2 – [512]	15	11	8	9	13	7
573	573	5/2 +, γ-vib	20	6	2	4	5	2
598			9	3	1			
644	643	11/2 1/2 – [521]	6	7	5	~ 1	5	6
665	668	{ 11/2 5/2 – [512] 5/2 5/2 – [523] }	11	10	11	31	38	22
711	711	9/2 +, γ-vib	7	7	2	5	7	3
750	753	3/2 3/2 – [521]	42	34	11	195	200	90
802	802	3/2 1/2 – [510]	255	~ 136	67	31	31	17
	812	5/2 5/2 + [642] ?				26	31	14
	843	9/2 5/2 – [523]				14	38	35
854		5/2 1/2 – [510]	73	~ 80	33			
894	895	7/2 3/2 – [521]	70	~ 77	23	150	200	110
	911	13/2 +, γ-vib				~ 6	~ 6	~ 1
	933	9/2 5/2 + [642]				42	60	34
941	943	7/2 1/2 – [510]	20	27	13	10	13	9
	967	11/2 5/2 – [523]				~ 1	3	2
	1002	9/2 3/2 – [521]				~ 2	4	3
1049	1052	11/2 11/2 – [505]	5			15	55	46
1084	1086	3/2 3/2 + [402]	21	15	9	242	345	215
	1109	13/2 5/2 + [642]				13	25	36
1132	1135	1/2 1/2 + [400]	46	39	18	269	384	224
1173			84	77	36			
	1190							26
	1205					8	21	24
	1222							12
1247			11	11	4			
1280			27	26	19			
	1302					44	6	7

(continued)

TABLE 5 (continued).

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
1332			2	3	2			
	1352					3	5	2
	1377	3/2 1/2 – [530]				62	75	47
1384		3/2 3/2 – [512]	123	67	28			
1408			5	10	5			
	1426					36	61	52
1440		5/2 3/2 – [512]	156	121	65			
	1525					~ 10	~ 9	9
1526		7/2 3/2 – [512]	70	74	45			
	1536					~ 13	~ 18	10
	1545					~ 12	~ 27	17
1548			76	49	17			
	1558						~ 26	~ 16
	1590					2	2	3
1596			88	53	29			
	1625					32	36	28
1629		9/2 3/2 – [512]	32	14	5			
	1638					6	11	6
1645			27	15	14			
	1657*					39	51	29
1684			126	71	30			
1718			268	178	80			
1747			202	126	60			
	1748					6	5	10
1779			15	13	7			
1800			15	15	6			
	1812					5	4	4
1815			34	24	13			
1842			30	28	15			
	1853					5	4	9
1865			222	122	60			
	1893					17	28	33
1912			194	98	48			
	1940					5	10	4

* Several weak groups from 1657 keV to 1892 keV.

TABLE 6. Levels populated in ^{169}Er.

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
0	0	1/2 1/2 – [521]	273	158	49	783	397	188
65	66	3/2 1/2 – [521]	~ 22	~ 16			~ 24	~ 17
74	74	5/2 1/2 – [521]	~ 50	~ 36	~ 22		~ 62	~ 37
90	91	5/2 5/2 – [512]	14	10	3	11	6	
176	176	7/2 5/2 – [512]	333	280	104	293	229	131
225	224	7/2 1/2 – [521]	116	96	37	212	167	95
	243	9/2 1/2 – [521]				9	11	25
285	284	9/2 5/2 – [512]	10	9	6	5	7	5
317	318	9/2 7/2 + [633]	12	16	5	67	54	31
415	414	11/2 5/2 – [512]	9	6	8	4	10	13
474	474	11/2 1/2 – [521]	7	7	8	7	11	23
527	527	13/2 7/2 + [633]	21	32	20	50	72	76
565		1/2 1/2 – [510]	~ 2	3				
599	599	3/2 1/2 – [510]	325	151		52	35	19
654	653	5/2 1/2 – [510]	115	83	38	30	25	11
714	713	3/2 3/2 – [521]	96		14	270	165	87
739	739	7/2 1/2 – [510]	36	29	21	9	7	
769	768	5/2 3/2 – [521] ?		2	1	4	6	
822		7/2 7/2 – [514]	33	41	22			
844	850	7/2 3/2 – [521]	35	44	23	154	132	97
930	927	9/2 7/2 – [514]	42	41	36	11	16	15
	940	7/2 5/2 – [523]				36	41	33
	~ 947	9/2 3/2 – [521]				9	~ 5	
	991					7	8	3
1051		11/2 7/2 – [514]	6	~ 3	3			
	1052	9/2 5/2 – [523]				33	30	23
	1076	11/2 3/2 – [521]				20	29	23
1082		3/2 3/2 – [512]	134	74	35			
	1096					7	7	
	1116					14	12	8
1119			25		7			
1141	1142	5/2 3/2 – [512]	208	167	80	11	10	7
1187	1186	11/2 5/2 – [523]	~ 5		~ 2	5	12	13
	1215					6	10	13
1230	1229	7/2 3/2 – [512]	101	71	47	26	25	22
	1239					16	19	10
	1274					7	6	5
1341		9/2 3/2 – [512]	7	~ 7	9			
	1360					42	54	38
1364			8	~ 7	4			

(continued)

Table 6 (continued).

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
1388			276	205	72			
	1394	11/2 11/2 – [505]				12	50	74
1415			72	63	29			
	1415						13	11
1457			18	12	6			
	1457					25	46	15
	1471					38	38	23
	1484					106	104	61
1488				362	119			
	1526[a]	3/2 3/2 + [402]				188	229	183
1535			74	67	39			
1554			145	95	38			
	1564					41	49	35
1570			163	64	25			
	1601					15	24	16
1608			106	69	34			
1622			66	52	28			
	1623					34		29
	1644	1/2 1/2 + [400]				151	199	140
1650			113	67	31			
	1677					107	109	95
1681				16	10			
1699				38	21			
	1702					36	31	34
1715				66	35			
	1718					9	31	34
1727				104	46			
1755			156	141	72			
1776			34	20	27			
	1790					33	43	39
1823			39	23	14			
	1825						7	11
1844			24	18	10			
	1857					50	69	74
1867			83	63	30			
	1886					10	9	
1899			31	26	12			
	1904					7	8	
1913			66	58	30			
	1924					52	24	19

TABLE 6 (continued).

Energy average		Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$			$d\sigma/d\Omega(d,t)$ $\mu b/sr$		
(d,p) keV	(d,t) keV		60°	90°	125°	60°	90°	125°
1929[b]			107	83	46			
	1958					9	10	
	1974					13	13	
	1994					6	8	
	2018					24	60	42
	2031						18	18
2053			147	120	46			
	2057						21	16
2092			32	17	12			
2123[c]			44	27	18			
2184			67	42	26			
2204			142	98	48			
2228			33	29	11			
2255			74	69	25			
2272			44	26	17			
2295			38	33	14			
2336			122	71	36			
2382				351	149			
2420				39	20			
2440				65	35			

a Unresolved groups from 1526 keV to 1564 keV.
b Unresolved groups from 1929 keV to 2053 keV.
c Unresolved groups from 2123 keV to 2184 keV.

11/2 – member is observed in ^{161}Er, ^{163}Er, and ^{169}Er. The 5/2 – member of the band is not predicted to be populated with an observable intensity. A weak group at the expected energy is observed in the ^{163}Er and ^{169}Er spectra, and may be caused by the Coriolis coupling between the 3/2 – [521] and the 5/2 – [523] orbitals.

In ^{169}Er, the 3/2 – [521] orbital is found a little lower than was the case in ^{167}Er. This lowering, which is unexpected in view of the usual rapid change in excitation energy with the neutron number, might be caused by admixtures of the gamma vibration based on the 1/2 – [521] ground state.

The absolute cross sections and relative values of $C^2_{j,l}$ obtained from the (d,t) reaction are given in Table 10. The intensities are considerably higher than the theoretical predictions, especially for the 3/2 – state in ^{165}Er (probably double, cf. Sec. 2.3.2) and the 7/2 – state in ^{163}Er. There are many

TABLE 7. Levels populated in ^{171}Er.

Energy (d, p)	Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$		
		60°	90°	125°
keV				
0	5/2 5/2 – [512]	22	10	3
76	7/2 5/2 – [512]	311	204	102
176	9/2 5/2 – [512]	~ 7		~ 9
195	1/2 1/2 – [521]	155	~ 66	21
~ 253	3/2 1/2 – [521]			~ 9
276	5/2 1/2 – [521]	48	29	24
304	11/2 5/2 – [512]	8	7	
378	9/2 9/2 + [624]	2	8	
420	7/2 1/2 – [521]	75	49	35
455	9/2 1/2 – [521]	10	7	6
531	7/2 7/2 – [514]	49	38	26
616	13/2 9/2 + [624]	28	32	30
645	9/2 7/2 – [514]	54	44	36
674	11/2 1/2 – [521]	5	4	3
706	1/2 1/2 – [510]	11	12	7
745	3/2 1/2 – [510]	552	287	143
795	5/2 1/2 – [510]	215	149	94
880	7/2 1/2 – [510]	105	70	50
906	3/2 3/2 – [512]	208	95	47
972	5/2 3/2 – [512]	211	161	114
1061	7/2 3/2 – [512]	100	71	55
1106	11/2 1/2 – [510]	6	4	12
1171	9/2 3/2 – [512]	10	9	9
1224		136	82	37
1261		53	32	15
1304		36	14	10
1376			314	150
1405			440	246
1435			24	31
1471		313	152	105
1508		165	102	67
1535		62	49	32
1570		138	94	51
1616		407	289	169
1647		46	41	20
1682		38	27	18
1722		64	52	35
1764		39	25	19
1795		236	170	97
1823		38		21

(*continued*)

Table 7 (continued).

Energy (d,p)	Assignment	$d\sigma/d\Omega(d,p)$ $\mu b/sr$		
		60°	90°	125°
keV				
1857		28	22	14
1925		119	112	46
1985			44	23
2093			70	40
2138			71	41
2172			48	27
2195			87	41
2265			187	80
2285			72	44
2308			44	23
2335			96	56
2361			350	204
2385			125	80

Table 8. Q-values and neutron separation energies for Er nuclei.

	$Q(d,t)$ A→A−1 keV	$Q(d,p)$ A−1→A keV	$S_n(d,t)$ keV	$S_n(d,p)$ keV
162	-2952 ± 10		9215 ± 10	
163		4682 ± 10		6907 ± 10
164	-2593 ± 10		8851 ± 10	
165		4431 ± 10		6657 ± 10
166	-2218 ± 10		8476 ± 10	
167		4214 ± 10		6439 ± 10
168	-1523 ± 10		7781 ± 10	
169		3781 ± 10		6006 ± 10
170	-1010 ± 10		7268 ± 10	
171		3458 ± 10		5683 ± 10

indications that these deviations are caused by Coriolis coupling to the several near-lying negative parity bands, but a quantitative explanation of the intensities must await a complete theoretical analysis based on the information now available on excitation energies and coupling matrix elements.

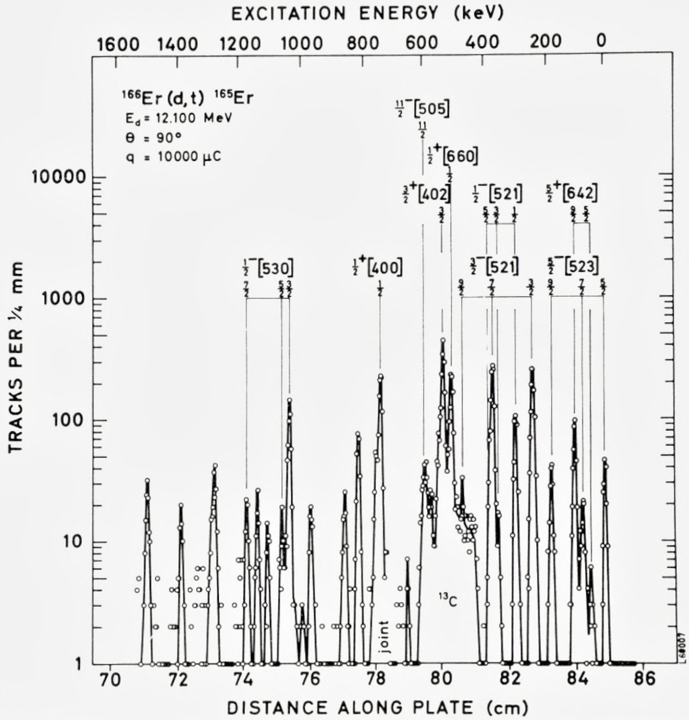

Fig. 4. Triton spectrum for the reaction ^{166}Er$(d,t)^{165}$Er $\theta = 90°$. In this and the following figures, groups ascribed to reactions on target impurities are indicated by the symbol of the target impurity. Thus, the broad group marked ^{13}C is due to the ^{13}C$(d,t)^{12}$C reaction.

2.3.2. The 5/2 + [642] Orbital

This orbital is the ground state in ^{161}Dy and should therefore be expected to appear as a low-lying state in ^{161}Er where, however, it has been impossible to identify the band with certainty. It is suggested that the strong state at 189 keV is the 9/2 + member of the band. The 13/2 + state can then be concealed in the strong groups around 369 keV to 396 keV or, more likely, in the too strong 7/2 5/2 – [523] group at 268 keV.

In ^{163}Er, the angular intensity variation for the strong group at 121 keV is consistent with a 9/2 5/2 + [642] assignment, although the cross section is somewhat large. The 22 keV 5/2 + level suggested by radioactivity studies[10] would fit into the band for an inertial parameter $A = 6$ keV. The weak group at 67 keV might be the 7/2 + state, and the 13/2 + state would then be expected near the strong 250 keV (7/2 3/2 – [521]) group, but it is not observed.

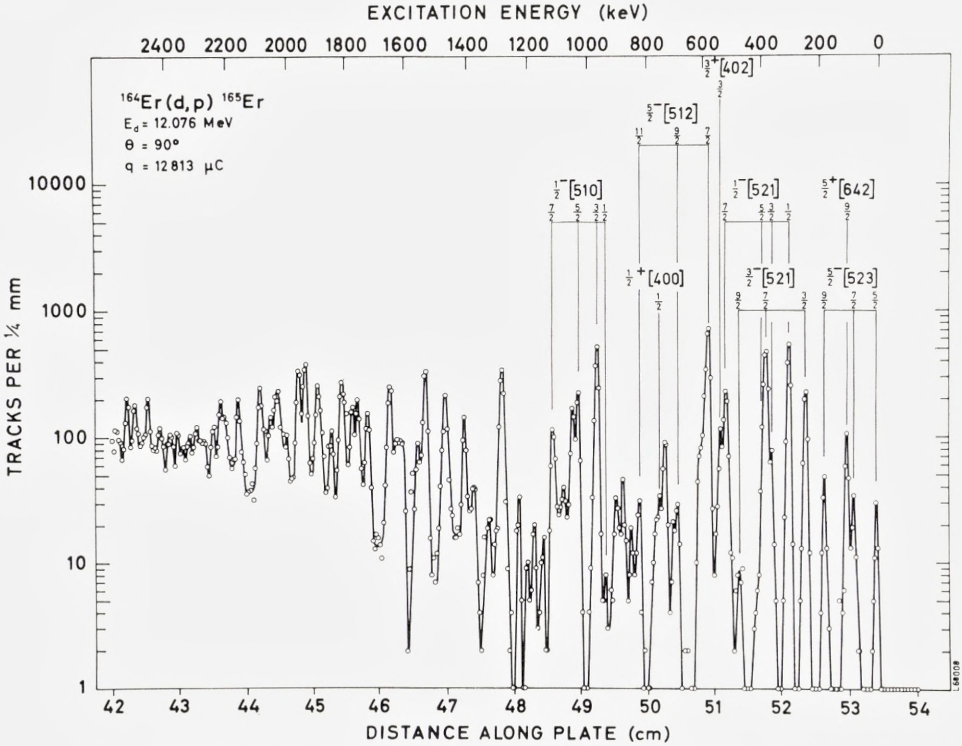

Fig. 5. Proton spectrum for the reaction ^{164}Er$(d,p)^{165}$Er $\theta = 90°$.

The 5/2 + [642] band poses problems also in ^{165}Er. A weak group is observed at 48 keV, which earlier was assigned as 5/2 5/2 + [642][10]. The only possible place for the 9/2 + group is 98 keV, and one could then suspect that the 13/2 + group is hidden in the 3/2 3/2 − [521] group at 242 keV, which is too intense. If this were the case, the band is distorted to a considerable extent.

The (d, t) spectra for ^{167}Er show two strong groups at 933 keV and 1109 keV with large 125° yields, which indicates high l-values. These groups might correspond to the 9/2 + and 13/2 + states of the 5/2 + [642] band. In view of the high excitation energies, the assignments are of course not very certain and are mentioned only because of the importance of observing the 3/2 + [651] (Sec. 2.3.3) and 5/2 + [642] orbitals in one nucleus.

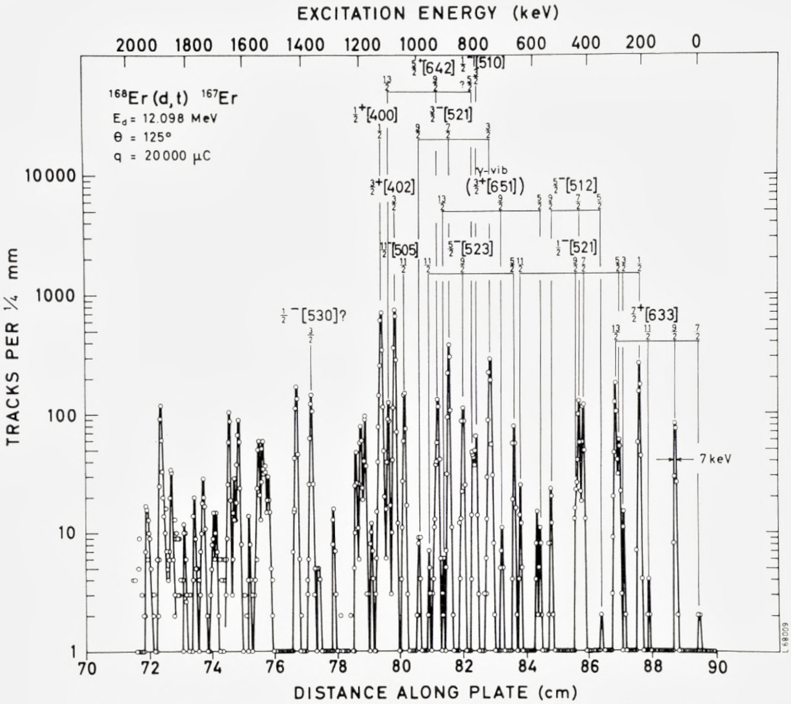

Fig. 6. Triton spectrum for the reaction ^{168}Er$(d,t)^{167}$Er $\theta = 125°$.

2.3.3. The 3/2 + [651] Orbital

The (d, t) spectrum of ^{161}Er contains three strong groups at 369 keV, 463 keV, and 481 keV, of which the two first have identical angular intensity variations which are different from that of the third group. In the present interpretation, the two lowest groups are associated with the 3/2 + [402] orbital (Sec. 2.3.6) which could interact strongly with the 3/2 + [651] orbital because of the crossing of these two states in the Nilsson diagrams and thus give rise to a splitting of the large 3/2 3/2 + [402] cross section. This phenomenon was observed earlier in ^{155}Gd[1]. The exact condition for the occurrence of a violent interaction between such crossing levels is not clear, but it could be strongly dependent on deformation[21]. It is remarkable that it is not observed for the 3/2 + levels in any of the other Er nuclei.

It has been suggested[11] that the $K = 3/2+$ gamma-vibrational band in ^{167}Er starts at 532 keV. Coulomb excitation[12] shows levels at 532 keV, 575 keV, and 642 keV. These levels are also observed in the (d, d') spectra[13]. In the (d,p) and (d, t) spectra, the 532 keV group cannot be resolved from

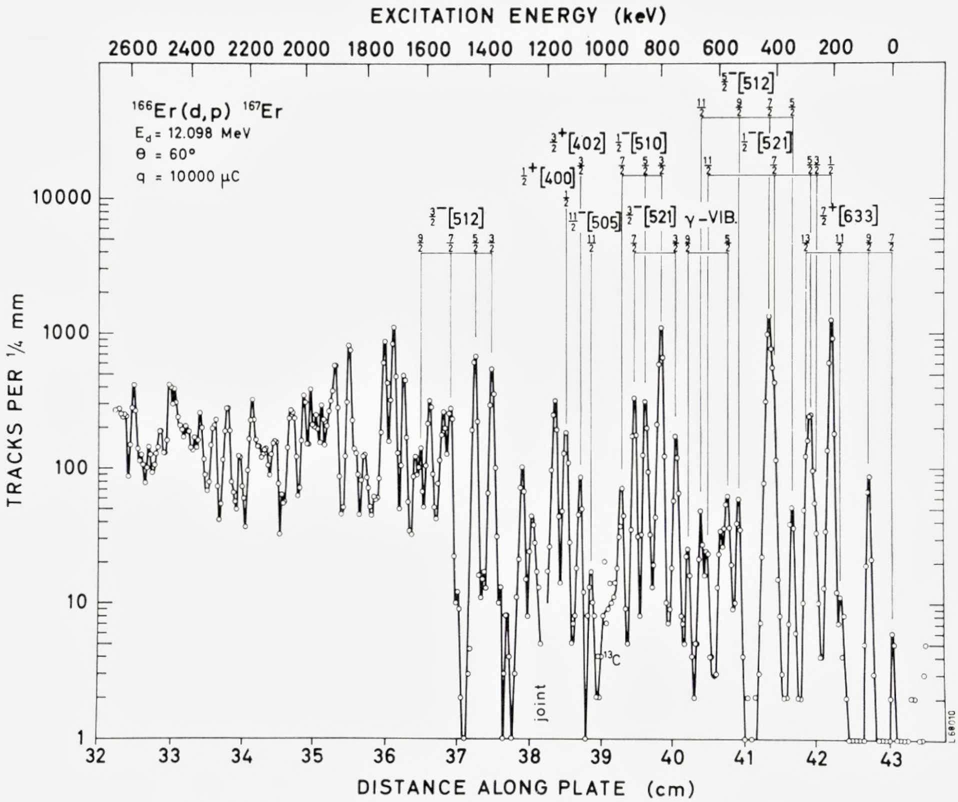

Fig. 7. Proton spectrum for the reaction ^{166}Er$(d,p)^{167}$Er $\theta = 60°$.

the 9/2 5/2 − [512] group, the intensity of which, however, shows that the 3/2+ contribution is low. This excludes any large admixture of the 3/2 3/2+[402] state into the gamma vibration (Sec. 2.3.6). A group in both the (d,p) and (d,t) spectra corresponds to the 575 keV level. The 643 keV level coincides with the 11/2 1/2 − [521] state, but levels at 711 keV and 911 keV could correspond to the 9/2 and 13/2 states in a $K = 3/2$ band. These groups are rather weak. One explanation for the (d,t) intensities to the vibrational band is an admixture of the 3/2+[651] wave function. The observed intensities are approximately 10 % of the theoretical prediction for a pure 3/2+[651] state. In judging this number, one should remember that the intensities observed for high l-transitions usually are somewhat larger than the calculated ones. The intensity of the 3/2+[651] component in the K-2 gamma vibration has theoretically[20] been estimated to 7 %. Probably there

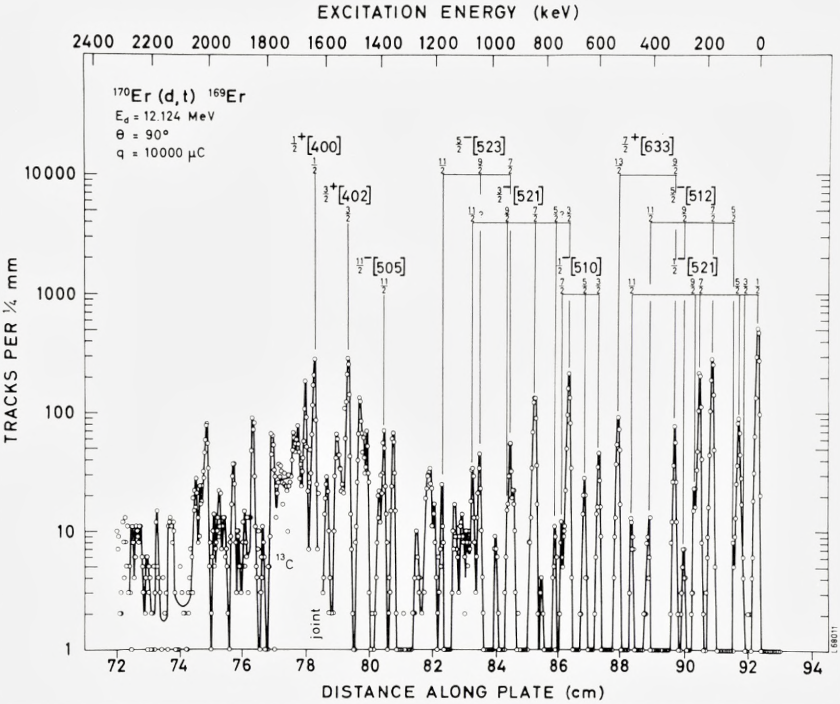

Fig. 8. Triton spectrum for the reaction ^{170}Er(d,t)^{169}Er $\theta = 90°$.

are other single-particle admixtures in the gamma vibration in ^{167}Er, as evidenced by the (d,p) population which hardly can be ascribed to the 3/2 + [651] hole state. Apart from the cases discussed above, it has not been possible to identify the 3/2 + [651] orbital in the Er isotopes. As in the Gd isotopes, the nonappearance of this orbital might be associated with its strong Coriolis coupling to the 5/2 + [642] orbital.

2.3.4. The 1/2 + [660] Orbital

The (d,t) cross section for the 1/2 1/2 + [400] state in ^{165}Er is approximately 25 % less than in ^{163}Er and ^{167}Er. A possible reason for this reduction in intensity is the coupling between the 1/2 + [660] and the 1/2 + [400] orbitals. The same phenomenon has been discussed for ^{159}Gd[1]. In order to find all triton groups in ^{165}Er with an $l = 0$ component, a measurement of the (d,t) spectra at 5° was performed. At this angle, the yields for all other l-values are low. In this way, the 742 keV group which is discussed below (Sec. 2.3.6) and the 507 keV group are singled out as belonging to 1/2 +

Table 9. Inertial parameters and decoupling parameters. Numbers in brackets are decoupling parameters for $K = 1/2$ bands.

Nilsson Orbital	171	169	167	165	163	161
3/2 – [521]		11.2	11.8	10.8	12.1	12.0
5/2 + [642]			7.3			
3/2 + [651]			8.5			
1/2 – [530]				10.2 (0.53)	8.9 (0.53)	
5/2 – [523]		12.4	11.0	11.0	11.9	13.6
7/2 + [633]		8.7	8.8			
1/2 – [521]	12.2 (0.68)	11.7 (0.85)	10.9 (0.72)	12.3 (0.56)	13.2 (0.41)	
5/2 – [512]	11.0	12.1	11.8	12.2	12.6	
7/2 – [514]	12.6	12.0				
9/2 + [624]	9.8					
1/2 – [510]	11.1 (0.10)	11.6 (0.067)	11.4 (0.087)	12.5 (– 0.01)	12.9 (– 0.32)	
3/2 – [512]	12.9	12.2	11.6	13.0		

Table 10. (d, t) population of the 3/2 – [521] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = -2$ MeV						Relative values of $C^2_{j,l}$					
	Theory	^{161}Er	^{163}Er	^{165}Er	^{167}Er	^{169}Er	Theory	^{161}Er	^{163}Er	^{165}Er	^{167}Er	^{169}Er
3/2	157	135	180	334	202**	139	0.10	0.10	0.10	0.23	0.19	0.23
5/2	0	–	~ 2*	–	–	5	~ 0	–	~ 0.003	–	–	~ 0.03
7/2	281	207	395	340	269	121	0.53	0.42	0.61	0.66	0.71	0.58
9/2	21	21	12	~ 9	6	5	0.25	0.27	0.12	0.11	0.10	0.16
11/2	9	17***	16	–	–	–	0.11	0.22	0.16	–	–	–

* From 60° yield.
** Assumes that 7/2 5/2 – [523] state contributes with 43 $\mu b/sr$.
*** Contains also the 9/2 5/2 – [523] state.

Table 11. (d, t) population of the 11/2 – [505] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = -2$ MeV					
	Theory	^{161}Er	^{163}Er	^{165}Er	^{167}Er	^{169}Er
11/2	82	71	95*	~ 62	84	66

* Contains also the 5/2 1/2 – [521] state.

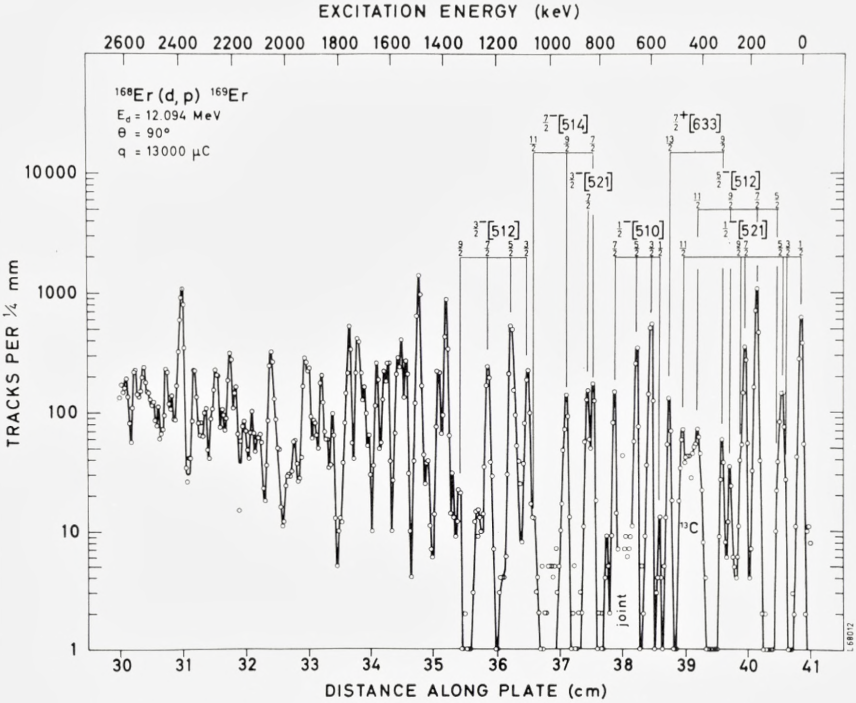

Fig. 9. Proton spectrum for the reaction ^{168}Er$(d,p)^{169}$Er $\theta = 90°$.

levels. The 507 keV level has already been assigned as 1/2+ from decay studies[10] and is probably associated with the 1/2 1/2+[660] state. If this interpretation is correct, the intensity corresponds to a 38% admixture of the 1/2 1/2+[400] state. It is not possible to identify with certainty other states of the 1/2+[660] band.

2.3.5. The 11/2-[505] Orbital

In the gadolinium isotopes, the 11/2−[505] orbital was observed between the 3/2−[521] and 3/2+[402] orbitals, and it is therefore expected to be observed also in the (d,t) spectra of erbium. A unique identification of the 11/2−[505] orbital is, however, rather difficult, because only the 11/2− member of the band is populated. On the basis of the angular dependence, a possible 11/2 11/2−[505] state has been located in all the erbium nuclei from ^{161}Er to ^{169}Er, but, in some cases, several groups with $l = 5$ angular dependence and reasonable intensity are present. The group with the lowest energy is then preferred.

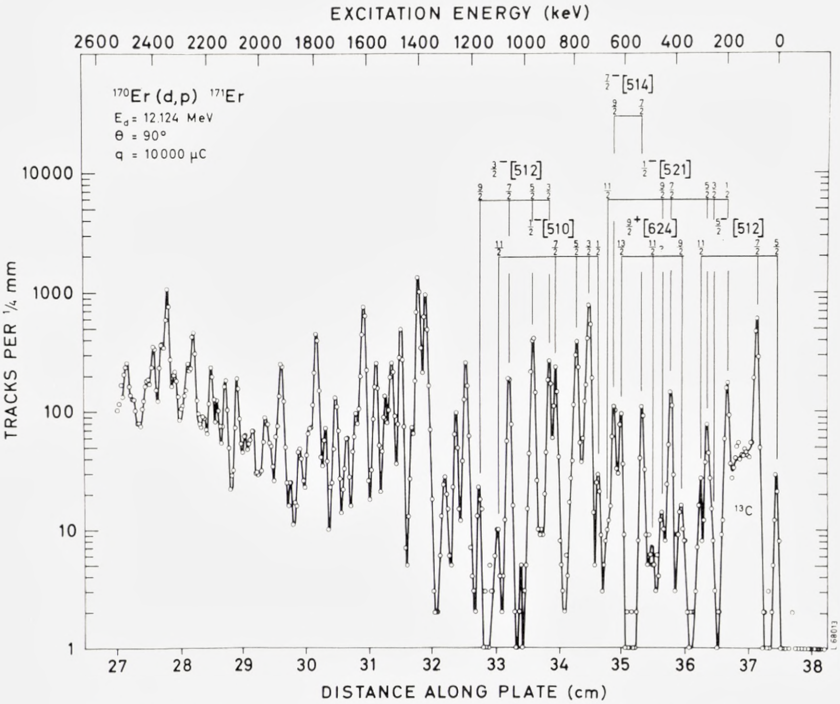

Fig. 10. Proton spectrum for the reaction $^{170}Er(d,p)^{171}Er$ $\theta = 90°$.

2.3.6. The 3/2 + [402] and the 1/2 + [400] Orbitals

The 3/2 + [402] and the 1/2 +400] orbitals are expected to give rise to intense groups in the (d,t) spectra. Two strong groups were observed in the gadolinium spectra[1)] and were ascribed to the 3/2 3/2 + [402] and the 1/2 1/2 + [400] states. In the erbium spectra similar groups are observed, and it is again reasonable to associate them with the two $N = 4$ states which originate in the $d_{3/2}$ and $s_{1/2}$ shell-model states. As in the gadolinium case, there are rather large fluctuations in intensities and problems with the assignment of the associated rotational bands, which make the distinction between the 3/2 + [402] and 1/2 + [400] orbitals difficult.

The (d,t) spectra recorded at 5° for ^{165}Er and ^{167}Er show that the upper level has $l = 0$, and this has been assumed to be the case for the other nuclei as well. Some additional support for the assignments was obtained from the angular intensity variations and the absolute cross sections.

The absolute cross sections, reduced to $Q = -2$ MeV, are given in Table 12 for the $N = 4$ orbitals and indicate an increased filling of the 3/2 3/2 + [402]

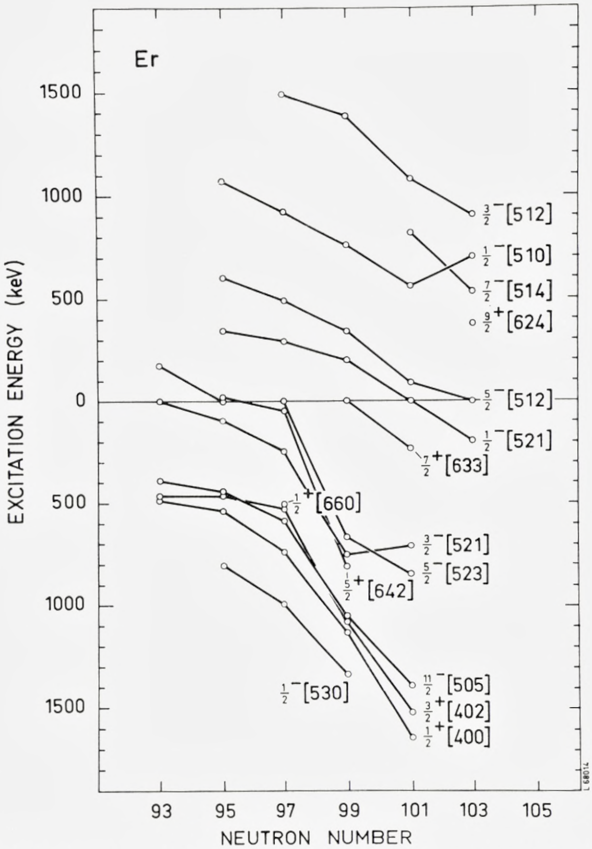

Fig. 11. Energies of the band heads for the Nilsson states observed. Points at negative energies indicate hole states.

state from ^{161}Er to ^{167}Er and a dilution of the state in ^{169}Er. The 1/2 1/2 + [400] group discloses the same general behaviour and, in addition, a somewhat reduced intensity in ^{165}Er (Sec. 2.3.4).

It has not been possible to identify the rotational bands built on the 1/2 + [400] and 3/2 + [402] states, although several of the rotational states are predicted to be populated quite strongly. The same situation prevailed in the Gd isotopes.

2.3.7. The 3/2-[532] Orbital

In spite of the relatively large (d, t) cross sections expected, no definite identification has been made of groups belonging to the 3/2 − [532] band.

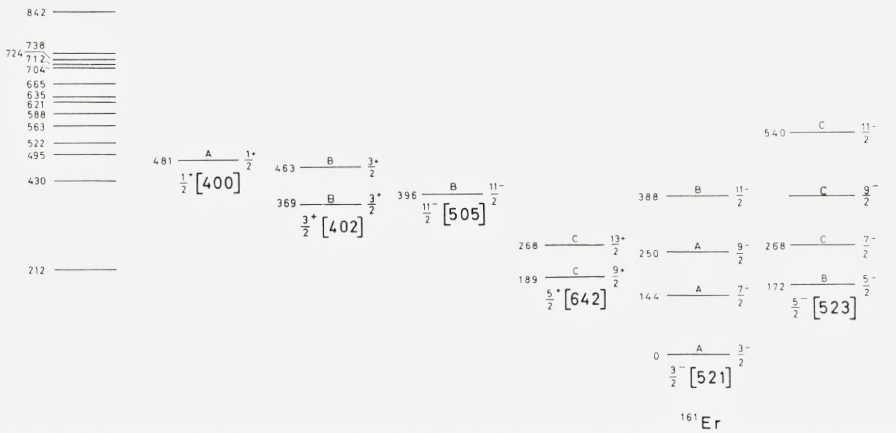

Fig. 12. Level scheme for ^{161}Er. Nilsson states to the left are hole excitations, those to the right are particle excitations. The letter A indicates that all the available data suggest the assignment, B an assignment consistent with the observations, but where lack of resolution or intensity prevents a definite assignment. Finally, C indicates that a group was observed at the position expected, e.g., for a rotational level but with an intensity considerably different from theoretically predicted intensity.

In ^{163}Er, there are several (d,t) groups between 600 keV and 800 keV of excitation, which might belong to this band, but no obvious rotational structure can be found. In ^{165}Er, there are unidentified groups around 1200 keV above the 1/2 – [530] band. In the Gd nuclei[1)], this energy region was free of strong lines, and it is conceivable that their presence in the heavier Er nuclei indicates that the 3/2 – [532] band has crossed the 1/2 – [530] band.

2.3.8. The 1/2–[530] Orbital

The strong (d,t) group at 856 keV in ^{163}Er has an angular variation which indicates a low angular momentum transfer, and as the intensity is close to the prediction for the 3/2 1/2 – [530] state, this identification is made, which is also supported by the analogy to the Gd nuclei where the band has been observed before[1)]. The groups at 877 keV and 973 keV are probably due to the 5/2 – and 7/2 – members of the rotational band. Their intensities are about 60 % of the theoretical predictions. The decoupling parameter is then $a = 0.53$ and the inertial parameter $A = 8.9$ keV, which is consistent with the findings in the Gd nuclei.

A similar band in ^{165}Er can be based on the strong (d,t) group at 1039 keV as the 3/2 1/2 – [530] group. Possible 5/2 – and 7/2 – groups are found at 1063 keV and 1172 keV. This band will have the same decoupling para-

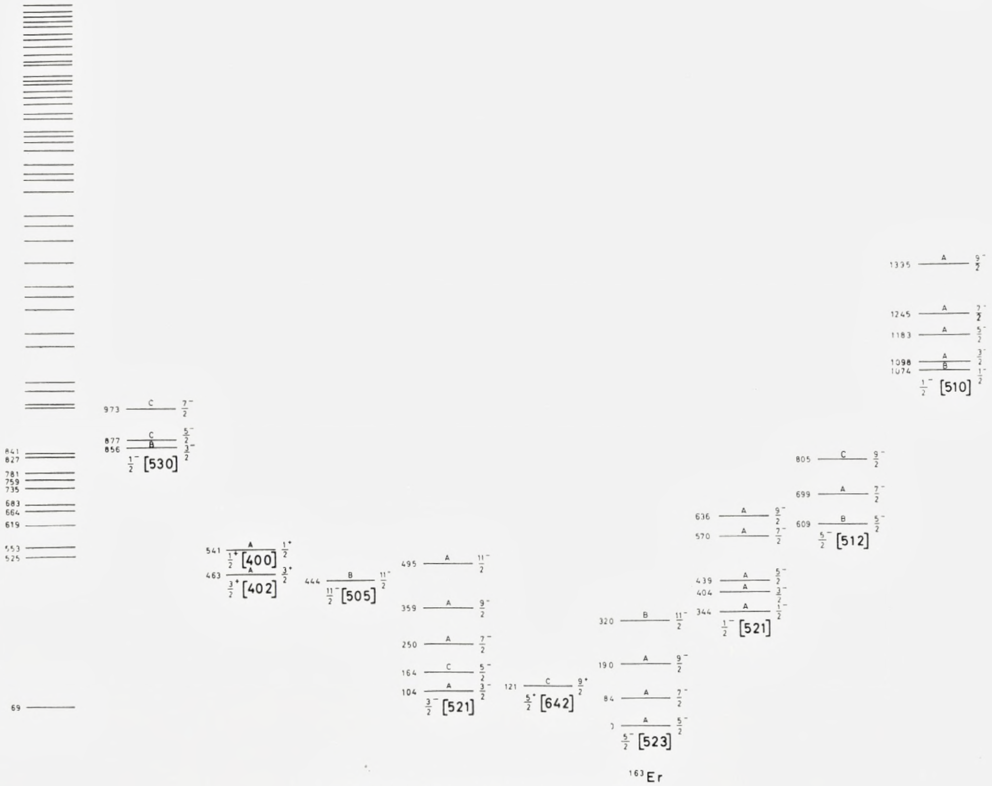

Fig. 13. Level scheme for ^{163}Er.

meter as in ^{163}Er, but $A = 10.2$ keV. Other choices for the 5/2 – and 7/2 – groups are, however, possible.

In ^{167}Er, the 3/2 1/2 – [530] state is probably the one at 1377 keV ($l = 1$?), but the associated rotational band is by no means obvious in the (d, t) spectra. The intensity of the 3/2 – group is 57 % of the theoretical intensity; this indicates a beginning breakdown of the 1/2 – [530] state, which seems to be complete in ^{169}Er where the state has not been located at all. It is interesting to note that the suggested 3/2 1/2 – [530] state in ^{167}Er is strongly populated in the (d, d') reaction[13], which would indicate admixtures of an octupole vibration.

TABLE 12. (*d,t*) population of the $N = 4$ states.

Level	$d\sigma/d\Omega$, $\theta = 90°$, $Q = -2$ MeV					
	Theory	^{161}Er	^{163}Er	^{165}Er	^{167}Er	^{169}Er
3/2 3/2 + [402]	612	366	450	567	600	346
1/2 1/2 + [400]	780	630	602	440	650	337

TABLE 13. (*d,t*) population of the 5/2 – [523] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = -2$ MeV					Relative values of $C^2_{j,l}$				
	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er
5/2	39	31	39	43	41***	0.07	0.11	0.11	0.10	0.11
7/2	41	~ 20*	19	43**	41	0.08	0.07	0.06	0.10	0.11
9/2	65	34	43	48	30	0.79	0.78	0.83	0.73	0.54
11/2	5	~ 2	–	4	12	0.06	0.05	–	0.06	0.23

* Estimated from 60° and 125° yields.

** Assumes that the unresolved 7/2 state has the same intensity as the 5/2 member of the band.

*** Assumes that the unresolved 5/2 state has the same intensity as the 7/2 member of the band.

TABLE 14. (*d,t*) population of the 7/2 + [633] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = -2$ MeV			Relative values of $C^2_{j,l}$		
	Theory	^{167}Er	^{169}Er	Theory	^{167}Er	^{169}Er
7/2	0.4	~ 1	–	0.001	0.002	–
9/2	25	43	37	0.07	0.08	0.07
11/2	0.6	~ 1	–	0.02	0.02	–
13/2	35	59	50	0.92	0.91	0.93

TABLE 15. (*d,p*) population of the 5/2 – [523] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = 3$ MeV			Relative values of $C^2_{j,l}$		
	Theory	^{163}Er	^{165}Er	Theory	^{163}Er	^{165}Er
5/2	44	17	9	0.07	0.07	0.07
7/2	45	29	10	0.08	0.11	0.07
9/2	63	29	16	0.79	0.82	0.86
11/2	5	–	–	0.06	–	–

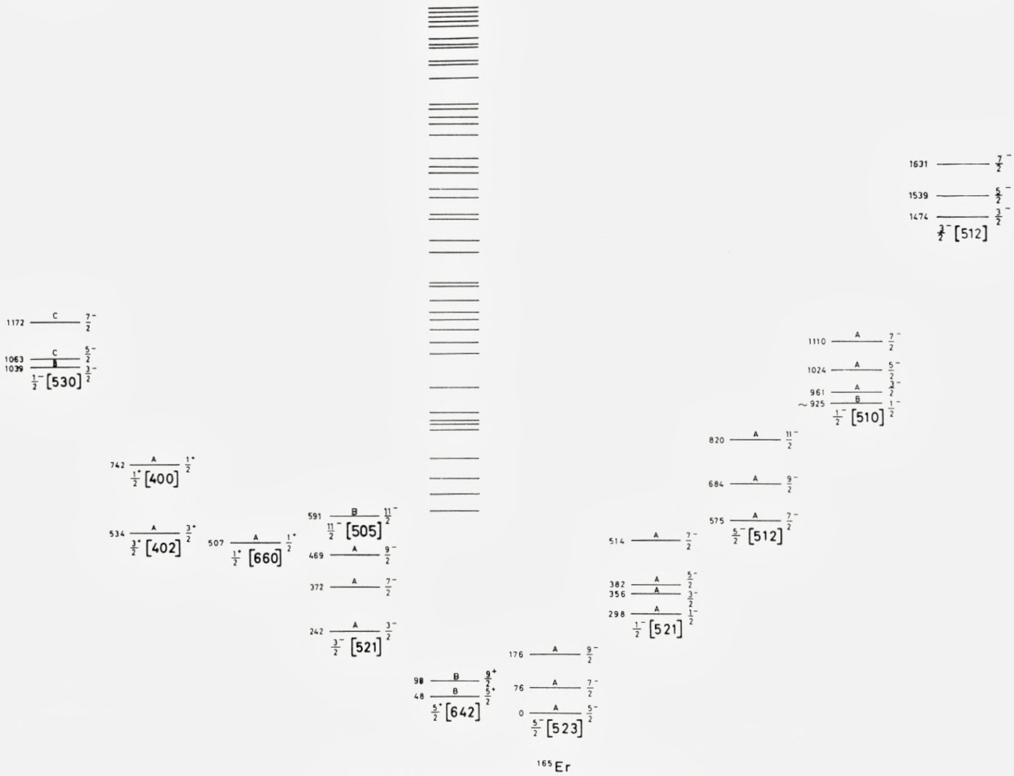

Fig. 14. Level scheme for ^{165}Er.

2.3.9. The 5/2–[523] Orbital

The level at 172 keV in ^{161}Er is assigned as 5/2 5/2 – [523], in agreement with decay studies[16)] which also place a 9/2 – state at 344.7 keV. This level is definitely not observed in the (d, t) spectra, in disagreement with the theoretical predictions for the 9/2 5/2 – [523] state. Therefore, the most reasonable band based on the (d, t) data is one where the band head is still placed at 172 keV, but where the 7/2 –group is a part of the strong group at 268 keV, and the 9/2 – group coincides with the 11/2 – state in the ground-state band at 388 keV. The 11/2 – member of the band could then be the group observed at 540 keV, or part thereof.

The 5/2 – [523] orbital forms the ground states in ^{163}Er and ^{165}Er, where all the members of the rotational band are observed, except the 11/2 – state in ^{165}Er which is obscured by the strong 1/2 1/2 – [521] group at 298 keV.

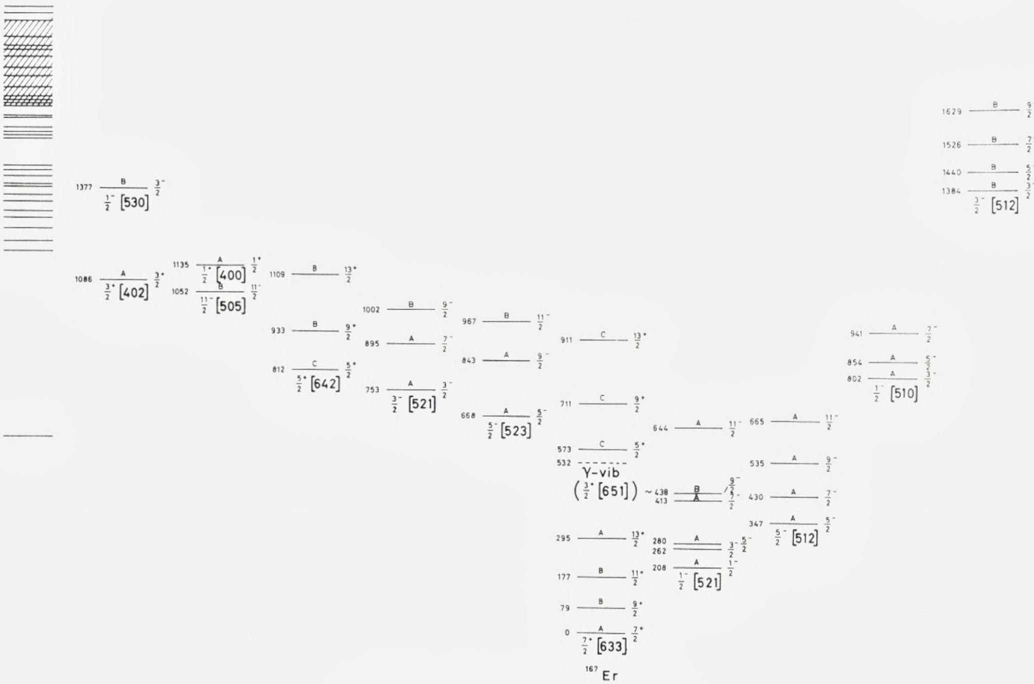

Fig. 15. Level scheme for ^{167}Er.

In ^{167}Er, a 5/2 5/2 – [523] level at 667.9 keV has been proposed earlier by Koch[14]. The (d,t) spectra show groups at 668 keV, 735 keV (coincides with the 3/2 3/2 – [521] state), 843 keV, and 967 keV, which could be the 5/2, (7/2), 9/2, and 11/2 states of this band. The intensities for the unobscured peaks are in good agreement with the theory.

In ^{169}Er, the 5/2 – [523] orbital is expected to be a component of the gamma vibration with $K = 5/2$ built on the 1/2 – [521] ground state. In the region between 900 keV and 1350 keV, there are approximately 13 weak lines in the (d,t) spectra, of which only a few have strong counterparts in the (d,p) spectra. The 5/2 – [523] band is expected in this region. The assignments for this band made from the ^{169}Ho decay[15] are 5/2 – at 850 keV and 7/2 – at 920 keV. If these assignments are accepted, the 5/2 – group is concealed in the 7/2 3/2 – [521] group and there is no 9/2 – group in the (d,t) spectra, which group is predicted to be as strong as the 5/2 – and 7/2 – groups. If the 7/2 – state is moved to 940 keV (which might be compatible with the decay data if the two final states for decay are shifted to

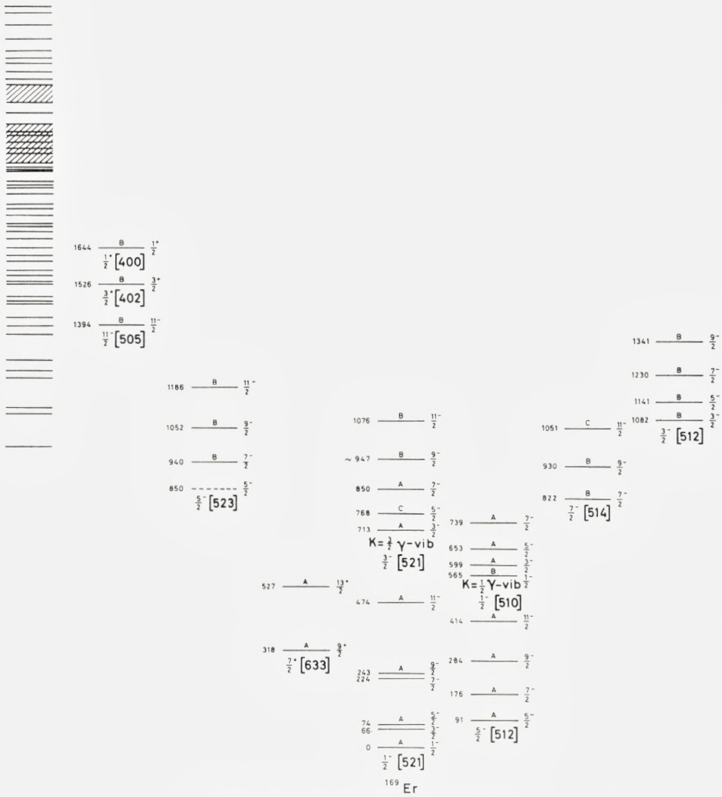

Fig. 16. Level scheme for ^{169}Er.

existing states approximately 20 keV higher), a 9/2 – group can be postulated at 1052 keV, where a (d, t) group is seen. The 11/2 – state can then be at 1186 keV. These assignments are given in Table 6.

2.3.10. The 7/2 + [633] Orbital

The states of the 7/2 + [633] ground-state band in ^{167}Er are observed up to the 13/2 + state. The (d, p) intensities are in reasonable agreement with the theory, whereas the (d, t) intensities for the 9/2 + and 13/2 + states are around 1.8 times the theoretical estimates. The partial filling of the ground-state level due to pairing can be estimated to reduce the cross sections by factors of U^2 and $V^2 \sim 0.5$ for the (d, p) and (d, t) reactions, respectively. The observed cross sections are thus considerably larger than expected and

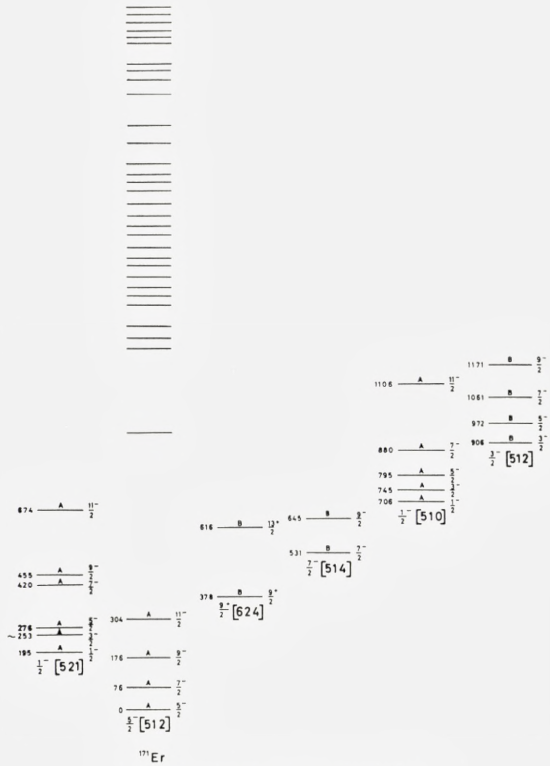

Fig. 17. Level scheme for ^{171}Er.

indicate a significant increase in single-particle strength. The strong Coriolis coupling between the $N = 6$ states might again be responsible for this effect.

In ^{169}Er, the 7/2+[633] band is placed with the 9/2+ state at 318 keV and the 13/2+ state at 527 keV. The 7/2+ group can then be expected to coincide with the 7/2 1/2−[521] group. The (d,p) intensities are approximately 0.8 times and the (d,t) intensities approximately 1.5 times the theoretical values.

It has not been possible to identify the band with any degree of certainty in the other nuclei.

2.3.11. The 1/2–[521] Orbital

This orbital has been identified in a large number of cases and is known in all the erbium nuclei[9]. The present results are in agreement with the previous assignments.

TABLE 16. (d,p) population of the $7/2+[633]$ band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = 3$ MeV			Relative values of $C^2_{j,l}$		
	Theory	^{167}Er	^{169}Er	Theory	^{167}Er	^{169}Er
7/2	~ 0.5	~ 0.5	–	0.001	~ 0.001	–
9/2	21	14	17	0.07	0.04	0.07
11/2	0.7	–	–	0.02	–	–
13/2	42	57	32	0.92	0.96	0.93

TABLE 17. (d,p) population of the $1/2-[521]$ band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = 3$ MeV						Relative values of $C^2_{j,l}$					
	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	^{171}Er	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	^{171}Er
1/2	377	164	181	213	214	73	0.25	0.17	0.21	0.20	0.20	0.15
3/2	38	37	19	14	~ 20	18*	0.02	0.04	0.02	0.01	0.02	0.03
5/2	107	50	39	52	~ 46	31	0.18	0.14	0.12	0.12	0.11	0.16
7/2	136	94	82	110	116	49	0.23	0.25	0.25	0.26	0.27	0.25
9/2	21	11	12**	16**	17**	7	0.27	0.22	0.26	0.27	0.29	0.27
11/2	4	9	~ 6	8	7	4	0.05	0.18	0.13	0.14	0.12	0.15

* From 125° yield.
** Assumes an intensity ratio 1:7 for the spin 7/2 and 9/2 members.

From Table 17 it is seen that the relative values of $C^2_{j,l}$ are in good agreement with the predicted values. The (d,t) and (d,p) cross sections for the states of the $1/2-[521]$ band show in a qualitative way the increased filling of the orbital from ^{167}Er to ^{171}Er, but the behaviour is less regular than that observed in the ytterbium isotopes[2] where the $1/2-[521]$ orbital showed all the characteristics of a pure single-particle state. Both cases differ from the gadolinium results, where large fluctuations in the cross sections were observed, probably as a result of the coupling to the gamma-vibrational states. The same couplings are possibly responsible for the slightly smaller $1/2-[521]$ intensity in ^{163}Er and ^{165}Er than in ^{167}Er and for the reduction of the decoupling parameters (cf. Table 9).

2.3.12. The 5/2–[512] Orbital

The $5/2-[512]$ orbital is characterized by a strong population of the $7/2-$ member of the band. It is the ground state in ^{171}Er[8] where all the members of the band are observed. In ^{169}Er and ^{167}Er, the band is expected as a low-lying particle excitation. The 176 keV group in ^{169}Er and the

430 keV group in ^{167}Er have the expected angular intensity variation. The 430 keV level has also been assigned to the 7/2 5/2 – [512] state from the ^{166}Er$(n,\gamma)^{167}$Er work[14]. The corresponding 5/2 –, 9/2 –, and 11/2 – states are all identified in the (d,p) spectra of ^{167}Er and ^{169}Er.

The theoretical cross sections and the reduced (d,p) cross sections are compared in Table 18. The 7/2 – state has a lower cross section than predicted, whereas the weakly populated 5/2 –, 9/2 –, and 11/2 – states are two to three times stronger than predicted.

In ^{165}Er, the strong 575 keV (d,p) group is assigned as 7/2 5/2 – [512], which assignment is supported by an $l = 3$ angular dependence. The intensity, however, corresponds to only 48 % of the expected cross section. The 5/2 – state is obscured, but the groups at 684 keV and 820 keV could be associated with the 9/2 – and 11/2 – levels, respectively.

In ^{163}Er, the levels at 609 keV, 699 keV, and 805 keV are possible candidates for the 5/2 –, 7/2 –, and 9/2 – states of the 5/2 – [512] band. The angular dependence agrees with the 7/2 – assignment for the 699 keV level, but the intensity is only 44 % of that expected.

A reduction in intensity of the 7/2 5/2 – [512] group in the lighter gadolinium nuclei is parallel to the one observed here.

2.3.13. The 7/2-[514] Orbital

This orbital is expected to appear at an excitation energy below 1 MeV for the heaviest erbium nuclei, in analogy to the assignments in ytterbium nuclei[2].

In ^{171}Er, the band has been placed at 531 keV (7/2 –) and 645 keV (9/2 –), but the reasons for the assignment are not compelling. The (d,p) groups selected occur in the expected energy region and have reasonable intensities and angular distributions.

There are three peaks in the (d,p) spectra of ^{169}Er with energies 822 keV, 930 keV, and 1051 keV, which can be associated with rotational members of the 7/2 – [514] band. The observed intensities of the 7/2 – and 9/2 – states are approximately equal, whereas the theory predicts a ratio of ~ 2 between the intensities of the 9/2 – and 7/2 – states. An alternative explanation for the states considered here would be an assignment to the 9/2 + [624] band, which is expected in the same region of energy.

2.3.14. The 9/2 + [624] Orbital

This orbital should occur as a particle excitation in the erbium nuclei. The pattern predicted consists of a weak 9/2 + group and a somewhat stronger 13/2 + group. In ^{171}Er, the band has been placed at 378 keV (9/2 +)

TABLE 18. (d,p) population of the 5/2 – [512] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = 3$ MeV						Relative values of $C^2_{j,l}$					
	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	^{171}Er	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	^{171}Er
5/2	6	9	–	13*	13	12	0.01	0.04	–	0.03	0.03	0.04
7/2	463	204	220	317	332	228	0.79	0.87	0.65	0.61	0.70	0.67
9/2	11	~ 3	8	13	11	~ 7*	0.14	0.09	0.17	0.18	0.17	~ 0.15
11/2	5	–	8	13	7	~ 7	0.06	–	0.17	0.18	0.11	~ 0.15

* From 60° yield.

TABLE 19. (d,p) population of the 1/2 – [510] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = 3$ MeV						Relative values of $C^2_{j,l}$					
	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	^{171}Er	Theory	^{163}Er	^{165}Er	^{167}Er	^{169}Er	^{171}Er
1/2	13	6	~ 2	–	~ 3	11	0.01	0.01	0.005	–	0.007	0.01
3/2	615	174	140	~157	160	262	0.40	0.38	0.35	0.35	0.35	0.31
5/2	172	50	53	~ 85	86	139	0.29	0.28	0.34	0.48	0.48	0.42
7/2	113	22	47	29	29	64	0.19	0.12	0.30	0.17	0.16	0.19
9/2	6	5	–	–	–	–	0.09	0.21	–	–	–	–
11/2	1	–	–	–	–	3	0.01	–	–	–	–	0.07

and 616 keV (13/2 +). The upper group has a rather flat angular distribution, and the 9/2 + level is an unassigned level at the expected energy. The inertial parameter is $A = 10.0$ keV, which can be compared to $A = 10.7$ keV in ^{175}Yb. The assignment must be considered somewhat uncertain, and it has not been possible to identify the orbital in the lighter erbium isotopes.

2.3.15. The 1/2–[510] Orbital

The 1/2 – [510] orbital has not been identified in the erbium isotopes before. It is characterized by a strong population of the 3/2 – state and somewhat smaller populations of the 5/2 – and 7/2 – states. The other members of the band are weak.

In ^{171}Er, all states from spin 1/2 to spin 11/2 in the 1/2 – [510] band are clearly observed. The 9/2 – state does, however, coincide with the 5/2 3/2 – [512] state. The intensities are approximately 60 % of the theoretical predictions, except for the 11/2 – state which is three times too strong.

In ^{169}Er, the 1/2 – [510] orbital is expected to have an excitation energy above 700 keV, but the most reasonable 3/2 – group is the one at 599 keV, which has an angular dependence in agreement with this assignment. The

Table 20. (d,p) population of the 3/2 – [512] band.

Spin	$d\sigma/d\Omega$, $\theta = 90°$, $Q = 3$ MeV					Relative values of $C^2_{j,l}$			
	Theory	^{165}Er	^{167}Er	^{169}Er	^{171}Er	Theory	^{167}Er	^{169}Er	^{171}Er
3/2	120	48	63	66	82	0.08	0.08	0.09	0.12
5/2	376	80	115	152	134	0.64	0.39	0.53	0.49
7/2	70	~ 27	68	63	57	0.12	0.23	0.22	0.21
9/2	11	–	12	6	7	0.15	0.29	0.15	0.18
11/2	1	–	–	–	–	0.01	–	–	–

Table 21. Comparison of experimental and theoretical single-particle amplitudes.

Nucleus	$K\pi[Nn_z\Lambda]$	Excitation energy		Decoupling parameter		% amplitude		Main vib. component theory
		exp	theor	a_{exp}	a_{theor}	exp	theor	
^{165}Er	1/2 – [521]	298	340	0.56	0.65	50	73	$Q(22)$ + 5/2 – [523) 22 %
^{167}Er	3/2 + [651]	325	750			10	7	$Q(22)$ + 7/2 + [633] 88 %
^{167}Er	1/2 – [510]	~ 768	800			30	32	$Q(22)$ + 5/2 – [512] 54 %
^{167}Er	3/2 – [521]	753	750			108	79	$Q(22)$ + 1/2 – [521] 15 %
^{169}Er	5/2 – [523]	~ 850	850			83	46	$Q(22)$ + 1/2 – [521] 47 %
^{171}Er	1/2 – [510]	706	800	0.10	– 0.17	52	48	$Q(22)$ + 5/2 – [512] 48 %

absolute intensity is less than in ^{171}Er. A lowering of the energy and the intensity is expected if the 1/2 – [510] state is a component of the $K = 2$ gamma vibration built on the 5/2 – [512] state.

In the lighter erbium isotopes, it is possible to find similar patterns which are ascribed to the 1/2 – [510] band. The intensities and the angular dependencies are in agreement with those observed in ^{169}Er.

The theory predicts a negative decoupling parameter ($a = -0.33$) for the 1/2 – [510] band. The decoupling parameters found in the erbium isotopes are all close to zero, in agreement with the expected effect of admixtures of gamma vibration based on the 5/2 – [512] state.

2.3.16. The 3/2–[512] Orbital

Relatively strong groups in the (d,p) spectra of ^{171}Er, ^{169}Er, ^{167}Er, and perhaps ^{165}Er form patterns which resemble the one expected for the 3/2 – [512] band. The intensities are, however, considerably smaller than the theoretical values, especially for the 5/2 – group, which has only about

35 % of the theoretical intensity compared to 50 % to 90 % for the other groups (cf. Table 20). A similar behaviour was observed in the Yb isotopes[2)], and there is therefore little doubt about the correctness of the assignments. The reason for the intensity reduction of the 5/2 – group is not clear, but it should be pointed out that none of the lower-lying bands has excessive 5/2 – strength.

3. Conclusions

The band-head energies of the Nilsson states identified in the erbium isotopes are shown in Fig. 11. The level order, with a few exceptions, is identical to the one found in Gd and Yb. Among the exceptions is the position of the 11/2 – [505] hole state, which in Gd always was found below the 3/2 + [402] state, but which in ^{165}Er is located at a higher excitation energy.

Most of the energy levels observed below 1 MeV of excitation have been explained in terms of the Nilsson model, although some of the observed cross sections deviate considerably from the theoretical prediction. Among the most noticeable discrepancies are those for the 3/2 3/2 – [521] and 7/2 3/2 – [521] states discussed in Sec. 2.3.1. As seen from the reduced (d,t) cross sections in Table 10, a spectroscopic factor for the 3/2 – state, defined as the ratio of the observed cross section to the calculated cross section, varies from 0.86 to 2.18 within a few mass numbers. A ratio of 1.0 would be expected for a pure hole state. The corresponding spectroscopic factors obtained from the (d,p) cross sections are of the order of 0.5, which is rather large for a hole state. Obviously, the description in terms of a pure Nilsson state is inadequate, but is not easy to find, e.g., sufficient $j = 3/2$ cross sections in the neighbouring bands to account for the observations.

A few of the Nilsson states expected in the region of low excitation have not been definitely observed. This is the case for the 3/2 + [651] state which has never been observed as a pure state, but which apparently is responsible for the irregular energy spacings and intensities in the 5/2 + [642] bands and also for the splitting of the 3/2 + [402] intensity observed in ^{161}Er. The 3/2 – [532] state offers another example of a state with a great tendency to fractionation and which, consequently, was not definitely observed in the Er nuclei. In this case, the responsible couplings have not been identified.

The energy spectra above 1 MeV of excitation are complex, and only for a few of the stronger groups it has been possible to make a single-particle assignment. Because of the Q-value dependence of the (d,t) cross section, the higher parts of the excitation spectrum can be studied only by the (d,p) reaction. Therefore, only states with large particle excitation components are accessible. Unfortunately, the energy resolution in the (d,p) spectra has not been sufficient for a more complete study of the regions of higher excitation energy. It is evident that strong couplings are active in spreading the intensity among several levels. The summed cross section is almost the same for all nuclei, but the level density is slightly decreasing with neutron number. This phenomenon is probably related to a decrease in collective strength, which is apparent from the inelastic deuteron scattering results[3)].

Several even-parity states are expected as particle states in the region above 1 MeV of excitation, among which the 1/2+[651] and the 1/2+[640] states have large cross sections. The first of these was observed at ~1700 keV in the heavier Gd nuclei and should be present in the Er nuclei as well. In ^{171}Er, there are several groups present around 1500 keV, which might belong to the 1/2+[651] band, but it has not been possible to identify a band structure. It should be remarked that strong even-parity states[17)] have been localized in the Yb nuclei by studies of isobaric analogue resonances. Some of these were erroneously ascribed to negative parity states before[2)].

In spite of the difficulties mentioned above, it is evident that the Nilsson model in general gives an amazingly accurate description of the low-lying energy levels in the Er nuclei. The level sequence is accurately reproduced and, in most cases, the components of the wave function obtained from the experiments are in good agreement with the theory.

A further improvement in the description of the states in odd deformed nuclei is obtained by specifically taking into account the particle-vibration interactions[19, 20)]. The experimental spectra show several effects of such interactions, but only in a few cases sufficient information is available to allow a closer comparison between theory and experiment. Table 21 summarizes the theoretical single-particle amplitudes for a number of cases, in which the coupling to the gamma vibrations has been considered theoretically[20)]. The corresponding experimental amplitudes have been obtained as the average ratios of the experimental and the calculated cross sections, as listed in Tables 10–20. The qualitative agreement between theory and experiment underlines the importance of this type of interaction for the low-energy spectra of odd nuclei.

The present work is meant to be a survey, and it has not been attempted to analyze the material in detail. A more complete test of the ideas underlying the description of the energy levels in deformed nuclei would be greatly facilitated by improved experimental data. Among the most obvious improvements is a better energy resolution, especially in the (d,p) spectra. The present techniques can be improved to ensure this. Better methods for l- and j-assignments would be invaluable. Frequently, the angular distribution studies performed up to now did not permit any unique assignments, but might be of increased value if they were combined with the parity information obtained from isobaric analogue resonance studies. Other interesting possibilities are connected with the study of excitation functions. A recent investigation[18] makes some promise that high and low angular momenta could be distinguished in this manner. An enhancement of the intensity of the high angular momentum groups could also be obtained by the use of the $(^3He,\alpha)$ pick-up reaction.

Acknowledgements

The authors wish to thank GUNVER DAMM JENSEN and ELSE NIELSEN for careful plate scanning. This work has greatly profited from the excellent targets prepared by G. SØRENSEN, I. THORSAGER, and V. TOFT at the Isotope Separator of the University of Aarhus.

The Niels Bohr Institute
University of Copenhagen

Physics Institute
University of Oslo

References

1) P. O. Tjøm and B. Elbek, Mat. Fys. Medd. Dan. Vid. Selsk. **36** (1967) no. 8.
2) D. G. Burke, B. Zeidman, B. Elbek, B. Herskind and M. C. Olesen, Mat. Fys. Medd. Dan. Vid. Selsk. **35** (1966) no. 2.
3) P. O. Tjøm and B. Elbek, Nuclear Physics **A 107** (1968) 385.
4) C. M. Perey and F. G. Perey, Phys. Rev. **132** (1963) 755.
5) M. Jaskola, K. Nybø, P. O. Tjøm and B. Elbek, Nuclear Physics **A 96** (1967) 52.
6) R. H. Siemssen and J. R. Erskine, Phys. Rev. **146** (1966) 911.
7) M. Jaskola and K. Nybø, private communication (1966).
8) Nuclear Data Sheets, National Academy of Science, Washington.
9) C. W. Reich and M. E. Bunker, unpublished.
10) B. Harmatz, T. H. Handley and J. W. Mihelich, Phys. Rev. **128** (1962) 1186.
11) L. K. Peker, Izv. Akad. Nauk. SSSR, Ser. Fis. **28** (1964) 289.
12) A. Tveter and B. Herskind, to be published in Nuclear Physics.
13) F. Sterba, P. O. Tjøm and B. Elbek, to be published.
14) H. R. Koch, Z. f. Phys. **187** (1965) 450.
15) L. Funke, H. Graber, K. H. Kaun and H. Sodan, Deutsche Akademie der Wissenschaften zu Berlin, Z. f. K. – Ph. **A 23**.
16) A. Abdumalikov, A. Abdurazakov and K. Gromov, Izv. Akad. Nauk. SSSR, Ser. Fis. **28** (1964) 257.
17) Y-t Liu and S. Whineray, to be published.
18) W. R. Hering and M. Dort, private communication.
19) D. R. Bès and Cho Yu-chung, Nuclear Physics **86** (1966) 581.
20) V. G. Soloviev and P. Vogel, Nuclear Physics **A 92** (1967) 449.
21) B. L. Andersen, Nuclear Physics **A 112** (1968) 443.

Indleveret til Selskabet den 17. januar 1969.
Færdig fra trykkeriet den 25. september 1969